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Admissible Controls in a Nonlinear Time-Optimal Problem with Phase Constraints ⋆

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Abstract: The paper is devoted to constructing admissible controls in a problem of optimal control by a nonlinear dynamic system under constraints on the current phase state. The dynamic system under consideration describes the controlled motion of a carrier rocket from the launching point to the time when the carrier rocket enters a given elliptic earth orbit. A problem consists in designing a program control for the carrier rocket that provides the maximal value of the payload mass led to the given orbit and the fulfillment of a number of additional restrictions on the current phase state of the dynamic system at the atmospheric part of the trajectory. The restrictions considered are due to the need to take into account the values of the dynamic velocity pressure, the attack angle and slip angle when the carrier moves in dense layers of the atmosphere. Such a problem is equivalent to a nonlinear time-optimal problem with phase constraints for carrier rockets of some classes. The algorithm for constructing admissible controls ensuring the fulfillment of additional phase constraints is suggested. The methodological basis of this algorithm is the application of some predictive control. This control is constructed in the problem without taking into account the constraints above. For a deterministic model of the atmosphere, such a predictive control is used to predict the values of a part of the phase state of the dynamic system at the next time. The prediction results are applied in the procedure of desired control construction. This procedure essentially takes into account specific features of the additional constraints. The results of numerical modeling are presented.

Keywords: dynamic system, iterative method, nonlinear control system, optimal control, predictive control, time-optimal control, phase constraints, admissible control.

1. INTRODUCTION

The problem of optimal carrier rocket launching to a given earth orbit is considered. It is required to construct a program control for the carrier rocket that provides the maximal value of the payload mass led to the given elliptic orbit. Taking into account a number of design features of some carrier types, this problem can be formulated as a time-optimal problem for a nonlinear dynamic system describing a controlled motion of some carrier rocket as a material point (the mass center) in a normal gravitational field (see Kostousov (2010)).

In the nonlinear time-optimal problem, a number of additional requirements are imposed on the current phase state of the dynamic system. The main attention is focused on dynamic constraints on the phase state of the system. These constraints are caused by the need to take into account the values of the dynamic velocity pressure, the attack angle and the slip angle when the carrier rocket moves in dense layers of the atmosphere.

An algorithm for constructing admissible controls in this problem is proposed. These controls ensure the fulfillment of restrictions on the product of the value of dynamic velocity pressure and the values of attack and slip angles

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at the atmospheric part of the the carrier rocket trajectory. The application of predictive control (see Eduardo (2007)) is the methodological basis of this algorithm. In the problem under consideration, such a control is a priori constructed without taking into account the dynamic constraints above. For a deterministic model of the atmosphere, such a predictive control is used to predict the values of a part of the phase state of the dynamic system at the next time. The prediction results are applied in the procedure of desired control construction. This procedure essentially takes into account specific features of the additional constraints.

Based on the informal sense of the optimal control problem, it is reasonable to use some control providing carrier rocket launching to the given orbit in a time close to the minimal one as a predictive control. This control can be constructed with the known methods (see, for example, Mazgalin (2010) and Kostousova (2010)). A similar approach based on the methodology for solving control problems with a guide within the framework of the scheme of N.N.Krasovskii and A.I.Subbotin (see Krasovskii (1974, 1985)) was used by the authors (see Kandoba (2016)) to construct admissible controls in the time-optimal problem for an essentially more complicated nonlinear dynamic system.

2. MATHEMATICAL MODEL OF THE CARRIER ROCKET CONTROLLED MOTION

Consider a nonlinear dynamic system describing the controlled motion of the mass center of the carrier rocket in a normal gravitational field from the launching time until the moment of its insertion into a given elliptic earth orbit. The motion of the rocket on the interval $[t_\mathrm{s}, t_\mathrm{f}]$ is described in some inertial rectangular coordinate system by the equations

$$\begin{aligned} &\dot{\boldsymbol{x}} = \boldsymbol{v}, \qquad \dot{\boldsymbol{v}} = W(t, \boldsymbol{x}, \boldsymbol{v}, \vartheta, \psi), \qquad \dot{m} = -\mu, \\ &\dot{\vartheta} = u_1, \qquad \dot{\psi} = u_2, \end{aligned} \tag{1}$$

where $\boldsymbol{x}, \boldsymbol{v} \in \mathbb{R}^3$ are the position and the velocity of the mass center of the rocket; $W(t, \boldsymbol{x}, \boldsymbol{v}, \vartheta, \psi)$ is the acceleration, which is given by the sum of the thrust acceleration, aerodynamic acceleration, and the gravitational acceleration (see Kostousov (2010)); m is the mass of the rocket; μ is a positive function describing the propellant consumption of the engine; ϑ and ψ are the pitch angle and yaw angle of the construction axle of the rocket, respectively; t_s is the time when the rocket starts its motion; t_f is the time when the rocket enters the required orbit.

The controls $\boldsymbol{u} \in \mathbb{R}^3$ are the rates u_1 and u_2 of change of the pitch angle ϑ and yaw angle ψ:

$$|u_1| \le u_1^{\max}, \qquad |u_2| \le u_2^{\max}. \tag{2}$$

The initial conditions for system (1) are given at the time t_s:

$$\begin{aligned} &\boldsymbol{x}_\mathrm{s} = \boldsymbol{x}(t_\mathrm{s}), \qquad \boldsymbol{v}_\mathrm{s} = \boldsymbol{v}(t_\mathrm{s}), \qquad m_\mathrm{s} = m(t_\mathrm{s}), \\ &\vartheta_\mathrm{s} = \vartheta(t_\mathrm{s}), \qquad \psi_\mathrm{s} = \psi(t_\mathrm{s}). \end{aligned} \tag{3}$$

The rocket must be inserted into a given osculating elliptic orbit with a given accuracy; i.e., the following terminal constraints must hold:

$$\begin{aligned} &|i - \bar{i}| \le \bar{\Delta}_i, \ |\Omega - \bar{\Omega}| \le \bar{\Delta}_\Omega, \\ &|h_{\min} - \bar{h}_{\min}| \le \bar{\Delta}_{h_{\min}}, \ |h_{\max} - \bar{h}_{\max}| \le \bar{\Delta}_{h_{\max}}, \\ &|\omega - \bar{\omega}| \le \bar{\Delta}_\omega, \end{aligned} \tag{4}$$

where the parameters defining the orbit are the inclination of the orbital plane i, the longitude of the ascending node Ω, the minimum height $h_{\min}$, the maximum height $h_{\max}$, and the argument of perigee ω. Here $\bar{i}$, $\bar{\Omega}$, $\bar{h}_{\max}$, $\bar{h}_{\min}$, $\bar{\omega}$ are the values of corresponding parameters of the given orbit and $\bar{\Delta}_i$, $\bar{\Delta}_\Omega$, $\bar{\Delta}_{h_{\max}}$, $\bar{\Delta}_{h_{\min}}$, $\bar{\Delta}_\omega$ are admissible deviations from these values.

A number of additional requirements are imposed on the controlled motion of the carrier rocket. These requirements lead to the initiation of a number of extra constraints on the current phase state of dynamic system (1). In particular, it is required that

$$|\vartheta(t)| \le \vartheta^{max}, \qquad |\psi(t)| \le \psi^{max}, \qquad t \in [t_\mathrm{s}, t_\mathrm{f}]. \tag{5}$$

Our goal is to design a program control $\boldsymbol{u}$, i.e., a measurable vector function $\boldsymbol{u}(t)$, $t \in [t_\mathrm{s}, t_\mathrm{f}]$, that satisfies condition (2) and takes the maximum mass of the carrier rocket to the specified orbit (i.e., it spends the minimum amount of the propellant) under all state constraints. Since the mass of the rocket m is independent of the control and decreases monotonically in time, the problem reduces to the following time-optimal problem.

Problem 1. For control system (1) with given initial conditions (3), find a program control $\boldsymbol{u}$ minimizing the value $J[\boldsymbol{u}(\cdot)] = t_\mathrm{f}$ and satisfying restrictions (2). Conditions (4) on the orbit parameters and constraints (5) must be also fulfilled.

Controls that satisfy in Problem 1 all the requirements except for the condition of optimality will be called admissible in this problem.

3. ALGORITHM FOR CONSTRUCTING ADMISSIBLE CONTROLS IN THE TIME-OPTIMAL PROBLEM WITH DYNAMIC CONSTRAINTS

The design features of carrier rockets of some types lead to the necessity to provide the fulfillment of some dynamic constraints when the carrier rocket moves in dense layers of the atmosphere. These constraints may include restrictions on the value of the dynamic velocity pressure $q(\boldsymbol{x}, \boldsymbol{v}) = \frac{1}{2}\rho(\|\boldsymbol{x}\|)\|\boldsymbol{V}\|^2$, the attack angle α and the slip angle β. Here, $\rho(\|\boldsymbol{x}\|)$ is the air density at the point $\boldsymbol{x}$; $\boldsymbol{V} = \boldsymbol{V}(\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{v}_w)$ is the vector of relative speed of the rocket, where $\boldsymbol{v}_w = \boldsymbol{v}_w(\boldsymbol{x})$ is the systematic wind speed (see Lebedev (1973)).

In particular, for some types of light-weight rockets, it is required that the values of the dynamic velocity pressure q, the attack angle α, and the slip angle β satisfy the inequalities

$$\begin{aligned} &|q(\boldsymbol{x}(t), \boldsymbol{v}(t))\alpha(t)| \le \bar{q}_{\alpha\beta}, \\ &|q(\boldsymbol{x}(t), \boldsymbol{v}(t))\beta(t)| \le \bar{q}_{\alpha\beta}, \end{aligned} \qquad t \in (\theta, \tau_1] \tag{6}$$

on an interval $(\theta, \tau_1]$ at the atmospheric part of the rocket trajectory. Here, $\theta > t_\mathrm{s}$ is the terminal time of the action of a control of a given structure; τ_1 is the time when the first stage of the rocket is discarded; $\bar{q}_{\alpha\beta}$ is the maximal admissible absolute value of the product of magnitude of dynamic velocity pressure and the attack angle (the slip angle).

Let us consider the case when the mathematical model of the atmosphere is deterministic. In addition, we assume that the functions $\rho = \rho(\|\boldsymbol{x}\|)$ and $\boldsymbol{v}_w = \boldsymbol{v}_w(\boldsymbol{x})$ are continuous.

Let $(t^{(n)}, t^{(k)}]$ be a time interval (for example $(t^{(n)}, t^{(k)}] = (\theta, \tau_1])$, where additional phase constraints (6) have to be satisfied in Problem 1. A procedure of sequential prediction of the value of the dynamic velocity pressure at next times is proposed. Such a prediction is carried out with the help of some admissible controls in Problem 1. These controls are determined by the phase state of system (1) at the current time. Below, these controls are called predictive.

The procedure is realized as follows. Let us consider some admissible control $\boldsymbol{u}$ in Problem 1 as a predictive control. This control is determined by the phase state $X(t^{(n)})$ of system (1) at the time $t^{(n)}$ and acts on the time interval $[t^{(n)}, t_f]$. Here, $X = (\boldsymbol{x}^\top, \boldsymbol{v}^\top, m, \vartheta, \psi)^\top \in \mathbb{R}^9$. Then, the grid $t^{(n)}{=}t_0{<}t_1{<}\ldots{<}t_N{=}t^{(k)}$, $t_{i+1}{=}t_i{+}\Delta t_i$, $i{=}0, 1, \ldots, N{-}1$, on the time interval $(t^{(n)}, t^{(k)}]$ is specified. The iterative procedure is started for t_i $(i = 0, 1, \ldots, N{-}1)$. On each iteration of this procedure at the current time t_i, the following operations are performed.

1. The given predictive control $\boldsymbol{u}$ takes dynamic system (1) from the current phase state $X(t_i)$ to the state $X(t_{i+1})$. If the phase state $X(t_{i+1})$ of system (1) satisfies dynamic constraints (6), then the desired control $\widetilde{\boldsymbol{u}}$ acting on the interval $[t_i, t_{i+1})$ is determined by the control $\boldsymbol{u}$: $\widetilde{\boldsymbol{u}}(t) = \boldsymbol{u}(t)$, $t \in [t_i, t_{i+1})$. Then, the next iteration of this procedure is performed.

Otherwise, the following operations are fulfilled.

2. For the phase state $X(t_{i+1})$, the values $q_{i+1} = q(\boldsymbol{x}(t_{i+1}), \boldsymbol{v}(t_{i+1}))$, $\Theta_{i+1} = \Theta(\boldsymbol{x}(t_{i+1}), \boldsymbol{v}(t_{i+1}))$ $\sigma_{i+1} = \sigma(\boldsymbol{x}(t_{i+1}), \boldsymbol{v}(t_{i+1})) \in (0, \pi/2)$ of the dynamic velocity pressure q, the pitch angle $\Theta{=}\arctan\left(\frac{V_2}{V_1}\right)$, the yaw angle $\sigma{=}-\arctan\left(\frac{V_3}{\sqrt{V_1^2{+}V_2^2}}\right)$ of the relative speed $\boldsymbol{V} = (V_1, V_2, V_3)^\top$ are calculated.

Maximal admissible absolute values $\bar{\alpha}_{i+1}$ and $\bar{\beta}_{i+1}$ of the attack angle α and of the slip angle β at the time t_{i+1} are determined with the help of inequalities (6) for the value q_{i+1}:

$$\bar{\alpha}_{i+1} = \frac{\bar{q}_{\alpha\beta}}{q_{i+1}}, \quad \bar{\beta}_{i+1} = \frac{\bar{q}_{\alpha\beta}}{q_{i+1}}.$$

3. In the rectangle $\Lambda = [-\bar{\alpha}_{i+1}, \bar{\alpha}_{i+1}] \times [-\bar{\beta}_{i+1}, \bar{\beta}_{i+1}] \subseteq \mathbb{R}^2$ (or in some its subset), the functions $\vartheta^p{=}\vartheta^p(\alpha, \beta)$ and $\psi^p{=}\psi^p(\alpha, \beta)$ are defined. The values of these functions specify the predicted values of the pitch angle ϑ and yaw angle ψ at the time t_{i+1} for admissible values α and β of the attack angle and of the slip angle, respectively. For this purpose, the relations

$$\begin{aligned} &\sin\alpha = -\cos(\sigma_{i+1})\sin(\Theta_{i+1} - \vartheta^p)/\cos(\beta), \\ &\sin\beta = \sin(\psi^p)\cos(\sigma_{i+1})\cos(\Theta_{i+1} - \vartheta^p) - \\ &\qquad \cos(\psi^p)\sin(\sigma_{i+1}) \end{aligned} \tag{7}$$

are used. These equalities can be obtained with the help of the known mathematical technique (see Siharulidze (1982); Apazov (1987)). The first equality in (7) implies

$$y(\alpha, \beta) = \frac{\sin(\alpha)\cos(\beta)}{\cos(\sigma_{i+1})}, \tag{8}$$

where $y(\alpha, \beta) = \sin(\vartheta^p(\alpha, \beta) - \Theta_{i+1})$. The second equality in (7) can be rewritten as a quadratic equation with respect to $z(\alpha, \beta) = \cos(\psi^p(\alpha, \beta))$. This equation has the two roots

$$\begin{aligned} z^{\pm}(\alpha, \beta) = &\frac{1}{\sin^2(\alpha)\cos^2(\beta) - 1}\big(\sin(\sigma_{i+1})\sin(\beta) \\ &\pm|\cos(\alpha)\cos(\beta)|\sqrt{D(\alpha, \beta)}\,\big), \end{aligned} \tag{9}$$

where

$$D(\alpha, \beta) = \cos^2(\sigma_{i+1}) - \sin^2(\alpha)\cos^2(\beta). \tag{10}$$

In Λ, equalities (8)-(10) formally define the four continuous functions $y = y(\alpha, \beta)$, $z^+ = z^+(\alpha, \beta)$, $z^- = z^-(\alpha, \beta)$, and $D = D(\alpha, \beta)$. It is easy to verify that $y(0, 0) = 0$, $D(0, 0) = \cos^2(\sigma_{i+1})$, and $z^-(0, 0) = \cos(\sigma_{i+1})$. For the value of the yaw angle σ_{i+1} of the relative velocity $\boldsymbol{V}$ at the time t_{i+1}, these magnitudes satisfy the inequalities:

$$|y(0, 0)| < 1, \quad D(0, 0) > 0, \quad |z^-(0, 0)| < 1.$$

Then, from the continuity of the functions $y = y(\alpha, \beta)$, $z^- = z^-(\alpha, \beta)$, and $D = D(\alpha, \beta)$, it follows that there exists a closed neighborhood $\Lambda^{(0)} \subseteq \Lambda$ of the point $(0, 0)$, where the functions above satisfy the conditions

$$\begin{aligned} &|y(\alpha, \beta)| \le 1, \quad D(\alpha, \beta) \ge 0, \\ &|z^-(\alpha, \beta)| \le 1, \quad \forall(\alpha, \beta) \in \Lambda^{(0)}. \end{aligned} \tag{11}$$

Thus, by virtue of (11), in $\Lambda^{(0)}$ the values of the functions y and z^- correctly determine the corresponding predicted values of the pitch angle $\vartheta^p(\alpha, \beta) = \Theta_{i+1} + \arcsin(y(\alpha, \beta))$ and of the yaw angle $\psi^p(\alpha, \beta) = \arccos(z^-(\alpha, \beta))$.

4. Admissible extreme values $\widetilde{\vartheta}^{\min}$, $\widetilde{\vartheta}^{\max}$ and $\widetilde{\psi}^{\min}$, $\widetilde{\psi}^{\max}$ of the pitch angle ϑ and of the yaw angle ψ at t_{i+1} are determined taking into account all the restrictions in Problem 1.

Let

$$\begin{aligned} \underline{\vartheta}^p &= \min_{(\alpha,\beta)\in\Lambda^{(0)}} \vartheta^p(\alpha, \beta), \quad \overline{\vartheta}^p = \max_{(\alpha,\beta)\in\Lambda^{(0)}} \vartheta^p(\alpha, \beta), \\ \underline{\psi}^p &= \min_{(\alpha,\beta)\in\Lambda^{(0)}} \psi^p(\alpha, \beta), \quad \overline{\psi}^p = \max_{(\alpha,\beta)\in\Lambda^{(0)}} \psi^p(\alpha, \beta), \end{aligned}$$

$$\begin{aligned} \underline{\vartheta} &= \vartheta(t_i) - u_1^{\max}\Delta t_i, \quad \overline{\vartheta} = \vartheta(t_i) + u_1^{\max}\Delta t_i, \\ \underline{\psi} &= \psi(t_i) - u_2^{\max}\Delta t_i, \quad \overline{\psi} = \psi(t_i) + u_2^{\max}\Delta t_i. \end{aligned}$$

In virtue of restrictions (2) and (5), if

$$[\underline{\vartheta}, \overline{\vartheta}] \cap [\underline{\vartheta}^p, \overline{\vartheta}^p] \cap [-\vartheta^{max}, \vartheta^{max}] \neq \emptyset,$$

$$[\underline{\psi}, \overline{\psi}] \cap [\underline{\psi}^p, \overline{\psi}^p] \cap [-\psi^{max}, \psi^{max}] \neq \emptyset,$$

then

$$\begin{aligned} \widetilde{\vartheta}^{\min} &= \max\{\underline{\vartheta}, -\vartheta^{max}, \underline{\vartheta}^p\}, \quad \widetilde{\vartheta}^{\max} = \min\{\overline{\vartheta}, \vartheta^{max}, \overline{\vartheta}^p\}, \\ \widetilde{\psi}^{\min} &= \max\{\underline{\psi}, -\psi^{max}, \underline{\psi}^p\}, \quad \widetilde{\psi}^{\max} = \min\{\overline{\psi}, \psi^{max}, \overline{\psi}^p\}. \end{aligned}$$

Otherwise, the iterative procedure is interrupted and the following conclusion is formulated. It is impossible to construct the admissible control $\widetilde{\boldsymbol{u}}$ providing the fulfillment of phase constraints (6) on the interval $(t^{(n)}, t^{(k)}]$ for a given value of $\bar{q}_{\alpha\beta}$ by the procedure in question on the base of the predictive control $\boldsymbol{u}$ in Problem 1.

5. A set $\widetilde{\Lambda}$ of elements $(\alpha,\beta)\in\Lambda^{(0)}$ specifying admissible pairs of the values of pitch and yaw angles at the time t_{i+1} is constructed. Let

$$\Lambda^{+}=\{(\alpha,\beta)\in\Lambda^{(0)}\mid \widetilde{\vartheta}^{\min}\le\vartheta^{p}(\alpha,\beta)\le\widetilde{\vartheta}^{\max},\ \widetilde{\psi}^{\min}\le\psi^{p}(\alpha,\beta)\le\widetilde{\psi}^{\max}\}.$$

Then, $\widetilde{\Lambda}$ is defined as a set of elements $(\alpha,\beta)\in\Lambda^{+}$, for which the phase state $X(t_{i+1}|\boldsymbol{u}^{+}(\cdot|\alpha,\beta))$ of system (1) satisfies phase constraints (6) at the time t_{i+1}. Here, the control $\boldsymbol{u}^{+}(t|\alpha,\beta)$ $(t\in[t_i,t_{i+1}))$ takes system (1) from the current phase state $X(t_i)$ to the state $X(t_{i+1}|\boldsymbol{u}^{+}(\cdot|\alpha,\beta))$ at the time t_{i+1}. Here, the control $\boldsymbol{u}^{+}(\cdot|\alpha,\beta)$ is given by the formulas

$$\begin{aligned}u_1^{+}(t_i,\alpha,\beta)&=(\vartheta^{p}(\alpha,\beta)-\vartheta(t_i))/\Delta t_i,\\ u_2^{+}(t_i,\alpha,\beta)&=(\psi^{p}(\alpha,\beta)-\psi(t_i))/\Delta t_i.\end{aligned}\tag{12}$$

If $\Lambda^{+}=\emptyset$ or $\widetilde{\Lambda}=\emptyset$, then the iterative procedure is interrupted and the conclusion given at stage 4 is formulated.

6. The desired control $\widetilde{\boldsymbol{u}}(t_i)=(\widetilde{u}_1(t_i),\widetilde{u}_2(t_i))$ acting on the interval $[t_i,t_{i+1})$ is constructed as follows. Let

$$(\alpha^{\star},\beta^{\star})=Arg\{\min_{(\alpha,\beta)\in\widetilde{\Lambda}}F(\alpha,\beta)\},\tag{13}$$

where

$$F(\alpha,\beta)=\|\boldsymbol{x}(t_{i+1}|\boldsymbol{u}^{+}(\cdot|\alpha,\beta))-\boldsymbol{x}(t_{i+1}|\boldsymbol{u}(\cdot)\|^{2}.\tag{14}$$

Then,

$$\widetilde{\boldsymbol{u}}(t)=\boldsymbol{u}^{+}(t|\alpha^{\star},\beta^{\star}),\quad t\in[t_i,t_{i+1}).\tag{15}$$

7. System (1) is transferred from the current phase state $X(t_i)$ to the phase state $X(t_{i+1})$ at the time t_{i+1} by the control $\widetilde{\boldsymbol{u}}$. After that, the time t_{i+1} is considered as the initial time for dynamic system (1). Then, initial conditions (3) for this system are initialized by the values of the corresponding components of its phase state $X(t_{i+1})$. An auxiliary optimal control problem, which is analogous to Problem 1, is considered on the interval $[t_{i+1},t_{\rm f}]$. Some admissible control $\boldsymbol{u}$ is constructed in this new problem. This control is used as a predictive control at the next iteration of the procedure.

If the iterative procedure is successfully completed, then the desired control $\widetilde{\boldsymbol{u}}(t)$ $(t\in[t_{\rm s},t_{\rm f}])$ on the interval $[t^{(k)},t_{\rm f}]$ is given by the current predictive control $\boldsymbol{u}(t)$ $(t\in[t^{(k)},t_{\rm f}])$.

The results of numerical simulation show that the admissible controls $\widetilde{\boldsymbol{u}}$ in Problem 1 can be constructed on the time interval $(t^{(n)},t^{(k)}]\subseteq(\theta,\tau_1]$ with help of this procedure on the base of some predictive controls $\boldsymbol{u}$. These controls ensure the fulfillment of additional dynamic constraints (6) on the current phase state of system (1) on the time interval $(t^{(n)},t^{(k)}]$ for the values $\bar{q}_{\alpha\beta}\ge\bar{q}^{min}_{\alpha\beta}(\boldsymbol{u},t^{(n)},t^{(k)})$.

In conclusion, we note that the extreme shift methodology (see Krasovskii (1974, 1985)) is used for constructing the designed control $\widetilde{\boldsymbol{u}}$ in the procedure above. This methodology is realized by means of relations (13)-(15). The application of this methodology is justified when suboptimal (or optimal) controls in problem 1 are used as predictive controls $\boldsymbol{u}$. In this case, the main goal of the investigation is taken into account. We should to construct a control providing the insertion of the carrier rocket to the given orbit in a time close to minimal under additional phase constraints (6).

4. RESULTS OF NUMERICAL SIMULATION

For the numerical approbation of the algorithm described in Section 3, the special control $\boldsymbol{u}^{b}$, which is admissible in Problem 1 was used as a predictive control. This control can be constructed using the known method on the final atmosphere-free part of the trajectory (see Mazgalin (2010)). This method is based on the decomposition of the rocket motion into three components: lateral, vertical and horizontal movements. A such decomposition allows us to reduce the solution of the initial time-optimal problem to the sequential solution of auxiliary optimal control problems of the for the lateral and vertical motions of the carrier rocket. It has been empirically proved (see Kostousov (2010)) that the control $\boldsymbol{u}^{b}$ on the final atmosphere-free part of the trajectory provides carrier rocket launching to the given orbit in a time close to minimal.

With some assumptions corresponding to real conditions of rocket launching, approach proposed in (Mazgalin (2010)) can be used to construct the control $\boldsymbol{u}^{b}$ on the atmospheric section of the trajectory of the carrier rocket motion. The controls $\boldsymbol{u}^{b}$ constructed under these assumptions and acting from certain times at the atmospheric section until the final moment, are used as predictive controls in the procedure described above.

A numerical experiment was carried out. For different values of $\bar{q}_{\alpha\beta}$ controls $\widetilde{\boldsymbol{u}}$ providing the fulfillment of additional phase constraints (6) on the time interval $(\theta,\tau_1]$ in Problem 1 were constructed. The results of numerical simulation show that, for a carrier rocket of some type, control $\widetilde{\boldsymbol{u}}$ can be constructed for the value of $\bar{q}_{\alpha\beta}$ not less than 29 $\frac{kN\cdot\circ}{\mathrm{m}^2}$.

Fig. 1 shows the functions $Q_\alpha(t|\boldsymbol{u})=|q(\boldsymbol{x}(t),\boldsymbol{v}(t))\alpha(t)|$ $(t\in(\theta,\tau_1])$. These functions describe the dynamics of the absolute values of the products of the dynamic velocity pressure and the attack angle. The dotted black line in Fig. 1 presents the graph of the function $Q_\alpha(t|\boldsymbol{u}^{b})$, where the control $\boldsymbol{u}^{b}(t)=\boldsymbol{u}^{b}(t|X(\theta))$ $(t\in[\theta,t_{\rm f}])$ is determined by the phase state $X(\theta)$ of system (1) at the time θ and is constructed in Problem 1 without taking into account phase constraints (6). The solid red line in Fig. 1 presents the graph of the function $Q_\alpha(t|\widetilde{\boldsymbol{u}})$. The thin horizontal dotted line in this figure indicates the value of $\bar{q}_{\alpha\beta}$.

Fig. 2 shows the graphs of the first control components of $\boldsymbol{u}^{b}$ and $\widetilde{\boldsymbol{u}}$, respectively. In this figure, the dotted black line presents the graph of the first control component of $\boldsymbol{u}^{b}$, the solid red line, of control $\widetilde{\boldsymbol{u}}$. The second components of these controls have similar structures.

In Problem 1, the time when the rocket entered the given orbit with the help of the control $\boldsymbol{u}^{b}$ was 513.96 seconds, whereas with the help of the control $\widetilde{\boldsymbol{u}}$ it was 5.7 seconds longer.

Numerical modeling was carried out with the software package “StO-RN” (Certificate of state registration of the computer program 2017662091) in the center of collective use of the N.N.Krasovskii Institute of Mathematics and

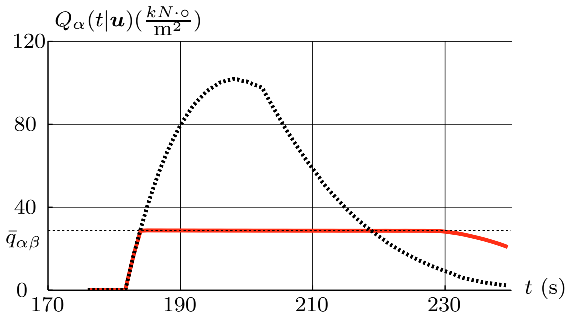

Fig. 1. The graphs of functions $Q_\alpha(t|\boldsymbol{u}^b)$ (dotted black line) and $Q_\alpha(t|\widetilde{\boldsymbol{u}})$ (solid red line) $t \in (\theta, \tau_1]$. $\bar{q}_{\alpha\beta} = 29\ \frac{kN\cdot\circ}{m^2}$.

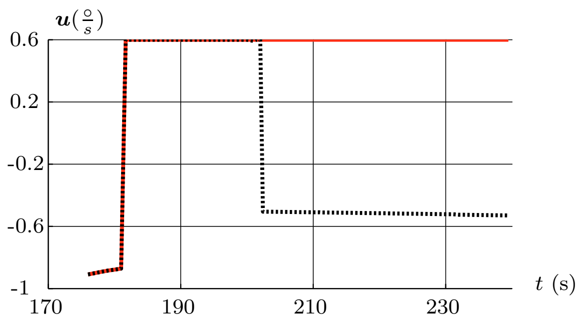

Fig. 2. The graphs of first components of controls $\boldsymbol{u}^b(t)$ (dotted black line) and $\widetilde{\boldsymbol{u}}(t)$ (solid red line) $t \in (\theta, \tau_1]$.

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5. CONCLUSION

The model predictive control technique used in the present paper has proved itself well for solving many applied control problems in the space industry (see, for example, Pascucci (2014), and Guiggiani (2015)). The results of numerical simulation using real data demonstrate the applicability of the proposed approach to construct admissible controls in the time-optimal problem for the nonlinear dynamic system. These controls ensure the fulfillment of the additional phase constraints on local time intervals.

The case of the deterministic mathematical model of the atmosphere is considered in the paper. In the case when the values ??of the atmosphere parameters can vary in some given ranges, the mathematical technique of the theory of differential games in the formalization of N.N.Krasovskii (see Krasovskii (1974)) can be used to construct admissible controls. The effectiveness of this approach is confirmed by the well-known experience (see Patsko (2009)) investigation of the problem of aircraft landing control under the conditions of wind disturbances without phase constraints. In this problem, the wind load is considered as a uncertain hindrance taking values ??in a given range. It seems that the mathematical statement of a similar problem of carrier rocket control in the atmospheric part of its movement and the development of solving methods using the theory of differential games are perspective directions for further research.

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